Chapter 6
Quadrilaterals

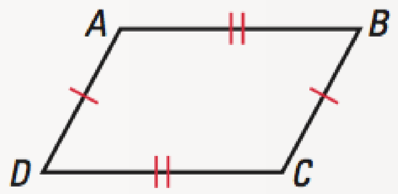
Section 3
Proving Quadrilaterals are Parallelograms

GOAL 1: Proving Quadrilaterals are Parallelograms

THEOREMS

THEOREM 6.6

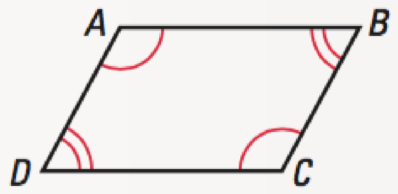
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



***ABCD* is a parallelogram.**

THEOREM 6.7

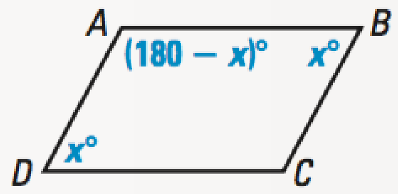
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



***ABCD* is a parallelogram.**

THEOREM 6.8

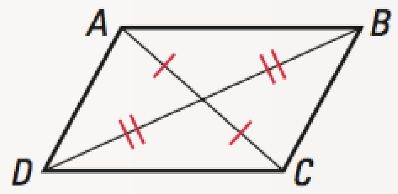
If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram.



***ABCD* is a parallelogram.**

THEOREM 6.9

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

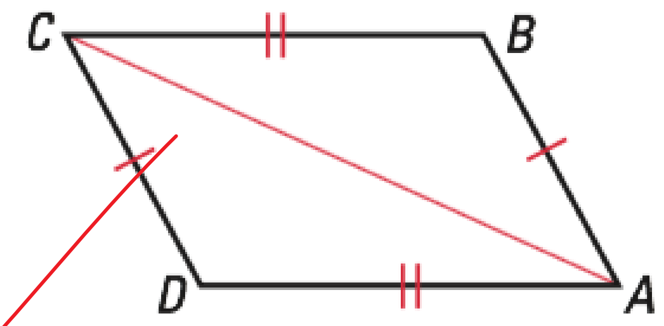


***ABCD* is a parallelogram.**

Example 1: Proof of Theorem 6.6

Given: $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$

Prove: ABCD is a parallelogram



Statements

1)

2)

3)

4)

5)

6)

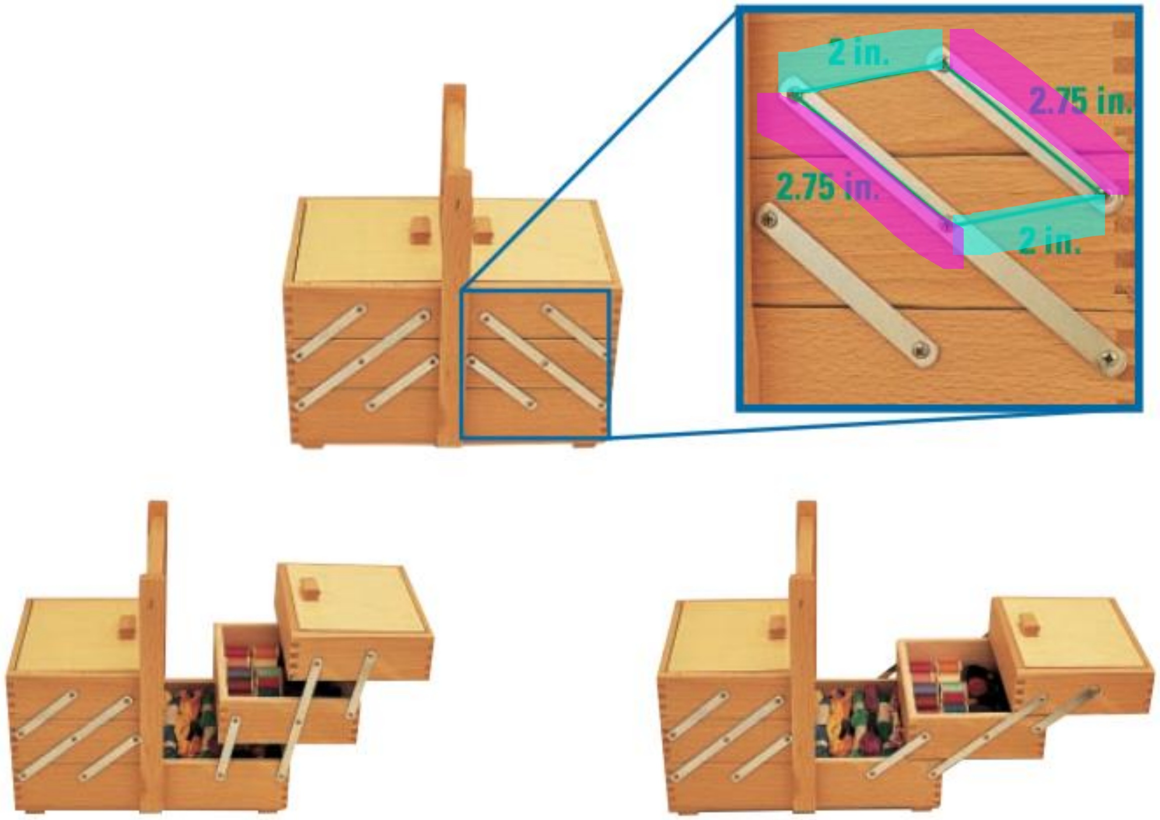
Reasons

Example 2: Proving Quadrilaterals are Parallelograms

As the sewing box below is opened, the trays are always parallel to each other. Why?

B/c both pairs of opposite sides are congruent, it is a parallelogram (6.6)

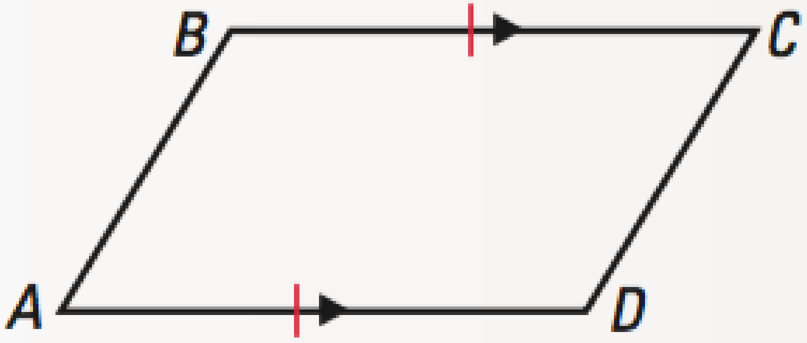
→ by def. both pairs of opposite sides are parallel



THEOREM

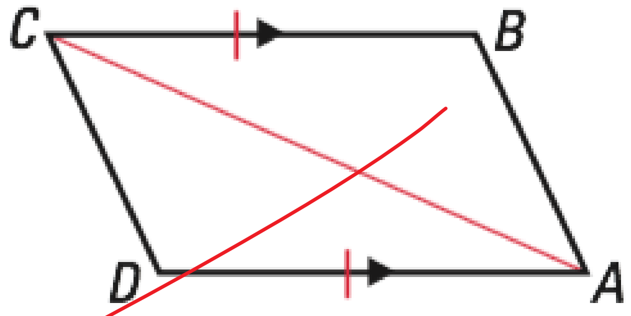
THEOREM 6.10

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.



***ABCD* is a parallelogram.**

Example 3: Proof of Theorem 6.10



Given: $\overline{BC} \parallel \overline{DA}$, $\overline{BC} \cong \overline{DA}$

Prove: ABCD is a parallelogram

Plan: Show that $\Delta BAC \cong \Delta DCA$, so $\overline{AB} \cong \overline{CD}$. Use Theorem 6.6.

Statements	Reasons
1)	
2)	
3)	
4)	
5)	
6)	

You have studied several ways to prove that a quadrilateral is a parallelogram. In the box below, the first way is also the definition of a parallelogram.

CONCEPT SUMMARY — **PROVING QUADRILATERALS ARE PARALLELOGRAMS**

- Show that both pairs of opposite sides are parallel. → slope
- Show that both pairs of opposite sides are congruent. → D.F.
- Show that both pairs of opposite angles are congruent.
- Show that one angle is supplementary to both consecutive angles.
- Show that the diagonals bisect each other.
- Show that one pair of opposite sides are congruent and parallel.

GOAL 2: Using Coordinate Geometry

When a figure is in the coordinate plane, you can use the Distance Formula to prove that sides are congruent and you can use the slope formula to prove that sides are parallel.

Example 4: Using Properties of Parallelograms

Show that A(2, -1), B(1, 3), C(6, 5), and D(7, 1) are vertices of a parallelogram.

Method 1: show opp. Sides are parallel

$AB \rightarrow \frac{3 - -1}{1-2} \rightarrow \frac{4}{-1} \rightarrow -4$

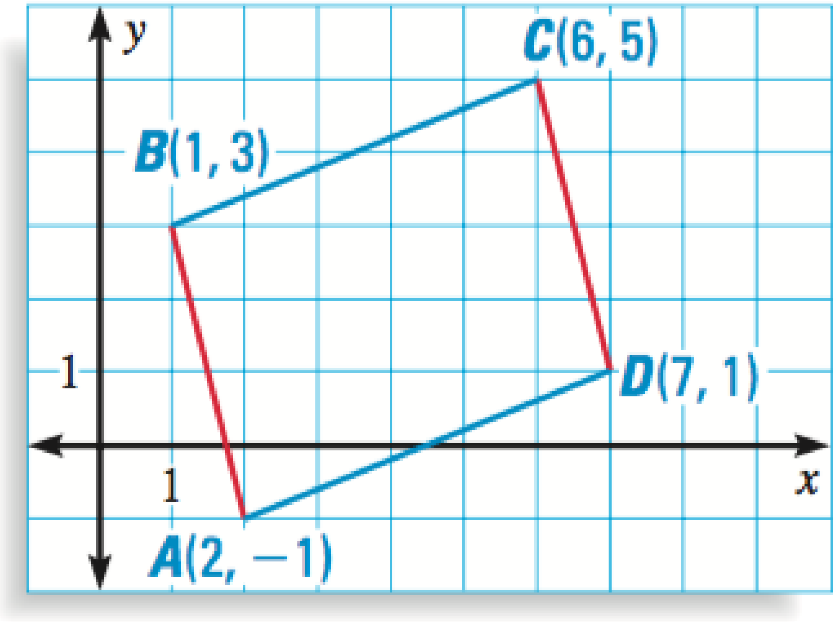
$CD \rightarrow \frac{5-1}{6-7} \rightarrow \frac{4}{-1} \rightarrow -4$

$AB \parallel CD$

$BC \rightarrow \frac{3-5}{1-6} \rightarrow \frac{-2}{-5} \rightarrow \frac{2}{5}$

$DA \rightarrow \frac{-1-1}{2-7} \rightarrow \frac{-2}{-5} \rightarrow \frac{2}{5}$

$BC \parallel DA$



Example 4 (continued)

A(2, -1), B(1, 3), C(6, 5), and D(7, 1)

Method 2: show opp. Sides are congruent

$$AB \to \sqrt{(2-1)^2+(-1-3)^2} \to \sqrt{1+16} \to \sqrt{17}$$

$$CD \to \sqrt{(6-7)^2+(5-1)^2} \to \sqrt{1+16} \to \sqrt{17}$$

$AB \cong CD$

$$BC \to \sqrt{(6-1)^2+(5-3)^2} \to \sqrt{25+4} \to \sqrt{29}$$

$$DA \to \sqrt{(7-2)^2+(1-{-1})^2} \to \sqrt{25+4} \to \sqrt{29}$$

$BC \cong DA$

EXIT SLIP